Quantization of adiabatic pumped charge in the presence of superconducting lead

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We investigate the parametric electron pumping of a double barrier structure in the presence of a superconducting lead. The parametric pumping is facilitated by cyclic variation of the barrier heights x_1 and x_2 of the barriers. In the weak coupling regime, there exists a resonance line in the parameter space (x_1, x_2) so that the energy of the quasi-bound state is in line with the incoming Fermi energy. Levinson et al found recently that the pumped charge for each pumping cycle is quantized with $Q = 2e$ for normal structure when the pumping contour encircles the resonance line. In the presence of a superconducting lead, we find that the pumped charge is quantized with the value $2e$.

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Physics of parametric electron pump has attracted great attention recently[1−12]. A classical example of electron pump is the Thouless pump facilitated by a traveling wave potential[13]. The pumped charge is quantized[13] and can be used as a quantum standard for electric charge.[14]. The quantization of pumped charge has also been studied for a large, almost open quantum dot[15,16] and a small, strongly pinched quantum dot[17]. In the latter case, there exists a resonance line along which the transmission through the quantum dot is at resonance. The pumped charge is quantized if the pumping contour in parameter space is properly chosen to encircle the resonance line[17]. Recently, we have studied the parametric pumping in presence of a superconducting lead[18]. At the normal conductor-superconductor (NS) interface, an incoming electron-like excitation can be Andreev reflected as a hole-like excitation[19]. In contrast to the current doubling effect[20], we found that due to the quantum interference of direct reflection and the multiple Andreev reflection, the pumped current is four times of the value when the leads are normal in the *weak pumping regime*. In this paper, we explore the effect of superconducting lead on electron pumping in the opposite limit, i.e., we study the pumped charge during the pumping cycle in the the strong pumping regime. Here the pumped charge equal to the pumped current multiplied by the period of pumping cycle. Similar to the Ref. 17, we examine the behavior of pumped charge near the resonance line. We find that the pumped charge in one pumping cycle is quantized with the value of $Q = 2e$ when one of the leads is superconducting.[21]

We consider a parametric pump which consists of a double barrier tunneling structure attached to a normal left lead and a superconducting right lead. Due to the cyclic variation of external parameters x_1 and x_2, the adiabatic charge transfer in the presence of a superconducting lead is[1,22,23]

$$Q^{NS} = 2e \int_0^\tau dt [\frac{dN_L}{dx_1}\frac{dx_1}{dt} + \frac{dN_L}{dx_2}\frac{dx_2}{dt}] \tag{1}$$

where τ is the period of cyclic variation and the quantity dN_L/dx is the injectivity[25,26] given, at zero temperature, by

$$\frac{dN_L}{dx_j} = \frac{1}{2\pi} Im[\mathcal{S}_{ee}^* \frac{\partial \mathcal{S}_{ee}}{\partial x_j} - \mathcal{S}_{he}^* \frac{\partial \mathcal{S}_{he}}{\partial x_j}] \tag{2}$$

where the first term is the injectivity of electron due to the variation of the external parameter[25,26], *i.e.* the partial density of states (DOS) for an electron coming from left lead and exiting the system as an electron, and the second term is the injectivity of a hole, *i.e.* the DOS for a hole coming from left lead and exiting the system as an electron. Using the Green's theorem, the pumped charge can be expressed as surface integral over area A enclosed by the path $(x_1(t), x_2(t))$ in the parameter space[1]

$$Q^{NS} = \frac{2e}{\pi} \int_A dx_1 dx_2 \Pi^{NS}(x_1, x_2) \tag{3}$$

with

$$\Pi^{NS}(x_1, x_2) = Im[\frac{\partial \mathcal{S}_{ee}^*}{\partial x_1}\frac{\partial \mathcal{S}_{ee}}{\partial x_2} - \frac{\partial \mathcal{S}_{he}^*}{\partial x_1}\frac{\partial \mathcal{S}_{he}}{\partial x_2}] \tag{4}$$

Note that the area A is a measure of variation of pumping parameters x_1 and x_2. A is very small in the weak pumping limit while remains finite in the strong pumping regime.

For the NS structures, the scattering matrix is described by 2×2 matrix $\hat{\mathcal{S}}$ when the Fermi energy is within the superconducting gap Δ.

$$\hat{\mathcal{S}} = \begin{pmatrix} \mathcal{S}_{ee} & \mathcal{S}_{eh} \\ \mathcal{S}_{he} & \mathcal{S}_{hh} \end{pmatrix} \tag{5}$$

where $\mathcal{S}_{ee}$ (or $\mathcal{S}_{he}$) is the scattering amplitude of the incident electron reflected as an electron (or a hole). Using Andreev approximation[19], we have[20,27]

$$\hat{\mathcal{S}} = \hat{S}_{11} + \hat{S}_{12}(1 - \hat{R}_I \hat{S}_{22})^{-1} \hat{R}_I \hat{S}_{21} \tag{6}$$

where $\hat{S}_{\beta\gamma}(E)$ $(\beta, \gamma = 1, 2)$ is a diagonal 2×2 scattering matrix for the double barrier structure with matrix element $S_{\beta\gamma}(E)$ and $S^*_{\beta\gamma}(-E)$. For instance, we have

$$\hat{S}_{11} = \begin{pmatrix} S_{11}(E) & 0 \\ 0 & S^*_{11}(-E) \end{pmatrix} \tag{7}$$

In Eq.(6) $\hat{R}_I = \alpha\sigma_x$ is the 2×2 scattering matrix at NS interface due to the Andreev reflection with off diagonal matrix element α. Here $\alpha = (E - i\nu\sqrt{\Delta^2 - E^2})/\Delta$ with $\nu = 1$ when $E > -\Delta$ and $\nu = -1$ when $E < -\Delta$. In Eq.(6), the energy E is measured relative to the chemical potential μ of the superconducting lead. Eq.(6) has a clear physical meaning[27]. The first term is the direct reflection from the normal scattering structure and the second term can be expanded as $\hat{S}_{12}\hat{R}_I\hat{S}_{21} + \hat{S}_{12}\hat{R}_I\hat{S}_{22}\hat{R}_I\hat{S}_{21} + ...$ which is clearly the sum of the multiple Andreev reflection in the hybrid structure. It is the quantum interference of these two terms which gives rise the enhancement of pumped current in the weak pumping regime for NS system[18]. From Eq.(6) we obtain the well known expressions for the scattering matrix $\mathcal{S}_{ee}$ and $\mathcal{S}_{he}$[20]

$$\mathcal{S}_{ee}(E) = S_{11}(E) + \alpha^2 S_{12}(E)S_{22}^*(-E)M_e S_{21}(E) \quad (8)$$

$$\mathcal{S}_{he}(E) = \alpha S_{12}^*(-E)M_e S_{21}(E) \quad (9)$$

and

$$M_e = [1 - \alpha^2 S_{22}(E)S_{22}^*(-E)]^{-1} \quad (10)$$

The double barrier structure which we consider is modeled by potential $U(y) = V_1\delta(y + a/2) + V_2\delta(y - a/2)$ where V_1 and V_2 are barrier heights which varys in a cyclic fashion to allow the charge pumping. For this system the retarded Green's function $G^r(y, y')$ can be calculated exactly[30]. This is done by applying the Dyson's equation regarding that any one of the δ-barrier is just a perturbation of the remaining system. This way $G^r(y, y')$ is obtained by applying Dyson's equation twice starting from the Green's function of the one-dimensional free space. With $G^r(y, y')$ we can calculate scattering matrix exactly from the Fisher-Lee relation[28]

$$S_{\beta\gamma} = -\delta_{\beta\gamma} + iv G^r_{\beta\gamma} \quad (11)$$

where $G^r_{\beta\gamma} = G^r(y_\beta, y_\gamma)$ and $v = 2k$ is the electron velocity in the normal lead. For normal structure, we have[17]

$$S_{11} = [1 - ix_2 - (1 + ix_1)\sigma^2]/D \quad (12)$$

$$S_{22} = [1 - ix_1 - (1 + ix_2)\sigma^2]/D \quad (13)$$

and

$$S_{12} = S_{21} = x_1 x_2 \sigma/D \quad (14)$$

where $D = -(1 - ix_1)(1 - ix_2) + \sigma^2$, $x_{1,2} = 2kV_{1,2}$, and $\sigma = \exp(ika)$. For the double barrier structure, the resonant tunneling is mediated by the quasi-bound state. When the energy of the incident electron is in line with the energy of the quasi-bound state the transmission coefficient reaches maximum. The energy of quasi-bound states can be determined either by looking at the pole of the scattering matrix[17] which works well in one dimension or by calculating the dwell time of the incident electron for two or three dimensional systems[29]. In the case of double δ barriers structure, the energy of quasi-bound state is given by[17] $E = E_r + \Delta E$ with $\Delta E = -(k_r/a)(x_1 + x_2)$ where $E_r = k_r^2 = (n\pi/a)^2$ is energy of the bound state when the system is isolated. This defines a resonance line $x_1 + x_2 = -\delta$ in parameter space (x_1, x_2) along which the transmission is at resonance[17]. Here $\delta < 0$ is the detuning of the Fermi energy from the bound state.

To show the quantization of charge transfer in the NS system, it is useful to recall the calculation of the normal case and make the comparison. In the normal case the charge transfer is given by[1,21]

$$Q^N = \frac{2e}{\pi}\int_A dx_1 dx_2 \Pi^N(x_1, x_2) \quad (15)$$

$$\Pi^N(x_1, x_2) = Im[\frac{\partial\mathcal{S}_{11}^*}{\partial x_1}\frac{\partial\mathcal{S}_{11}}{\partial x_2} + \frac{\partial\mathcal{S}_{12}^*}{\partial x_1}\frac{\partial\mathcal{S}_{12}}{\partial x_2}] \quad (16)$$

The pumped charge in this case has been calculated in Ref. 17. In the *weak pumping limit*, it is easy to show that only $\partial_x S_{11}$ contributes to the pumped charge. In the strong pumping regime, we will show in the following that the contribution from $\partial_x S_{12}$ to the pumped charge in normal structure is zero. As discussed in detail in Ref. 17, we neglect the smooth energy dependence of x_1 and x_2. From Eq.(16), we obtain the contribution due to $\partial_x S_{12}$

$$\Pi_1^N(x_1, x_2) = F_1(x_1, x_2)/F_2^2(x_1, x_2) \quad (17)$$

with

$$F_1(x_1, x_2) = -2x_1 x_2(x_1 - x_2)\sin^2(\delta/2) \quad (18)$$

$$F_2(x_1, x_2) = x_1^2 x_2^2 + (x_1 + x_2)^2 + 2(x_1 + x_2)\sin\delta + 2(1 - x_1 x_2)(1 - \cos\delta) \quad (19)$$

To compute the surface integral of Π_1^N in Eq.(17), it is convenient to change the variables from $x_{1,2}$ to p and z:

$$x_1 = -p\ \delta\ (1 + z)/2 \quad (20)$$

and

$$x_2 = -p\ \delta\ (1 - z)/2 \quad (21)$$

with $0 < p < \infty$ and $-1 < z < 1$. Substituting Eqs.(20) and (21) into Eqs.(18) and (19) and expanding Eqs.(18) and (19) in terms of small δ, we have

$$F_1 = z(1 - z^2)\delta^5 p^3/8 \quad (22)$$

$$F_2 = \delta^2[(1 - p)^2 + \delta^2 g(p, z)] \quad (23)$$

where $g(p,z)$ (an even function of z) is given in Eq.(8) of Ref. 17. Since F_1 is an odd function of z, the contribution due to $\partial_x S_{12}$ to the pumped charge is zero.

Now we follow the same procedure to calculate the pumped charge for the NS system. For the parametric pumping at zero temperature, we only need the scattering matrix at the Fermi level, i.e., at $E=0$. From Eqs.(9) and (10), we see that $\mathcal{S}_{he}$ is a real quantity and hence makes no contribution to the pumped charge in Eq.(3). It is straightforward to calculate Π^{NS} using Eq.(8), from which we obtain, $\Pi^{NS}(x_1,x_2) = F_3(x_1,x_2)/F_4^3(x_1,x_2)$ where $F_3 = 4x_1^4x_2^3(2-2\cos\delta + x_2\sin\delta)$ and $F_4 = x_1^2x_2^2+2(x_1+x_2)^2+4(x_1+x_2)\sin\delta+4(1-x_1x_2)(1-\cos\delta)$.

In Fig.1 we plot both Π^{NS} and Π^N as well as their cross-sections along and perpendicular to the resonance line. We see that Π^{NS} and Π^N are peaked around the resonance line. Two features are worth noticing. First of all, the peak of Π^{NS} is much sharper than that of Π^N. This is understandable and is due to the resonance nature of NS structures near the resonance line. In the Breit-Wigner form, the transmission coefficients for normal and NS structures are, respectively $|S_{21}|^2 = \Gamma_1\Gamma_2/[(E-E_r)^2+\Gamma^2/4]$ and[20] $|\mathcal{S}_{he}|^2 = 4\Gamma_1^2\Gamma_2^2/[4(E-E_r)^2+\Gamma_1^2+\Gamma_2^2]^2$, where E_r is the resonant level, Γ_1 and Γ_2 are the decay widths into the left and right lead. Hence $|\mathcal{S}_{he}|^2$ decays much faster away from E_r than $|S_{21}|^2$. The scattering matrix S_{21} and $\mathcal{S}_{he}$ will appear, respectively, in Eqs.(3) and (15) implicitly as can be seen from Fisher-Lee relation Eq.(11) and the Dyson equation $\partial_{X_2}G_{11}^r = G_{12}^rG_{21}^r$[31]. Secondly, the peak height of Π^{NS} is four times larger than that of Π^N. This is precisely due to the constructive interference of direct reflection and multiple Andreev reflection[18]. Now the physics of pumping at resonance is clear. For the resonance pumping in the weak pumping regime, we are looking at the small neighborhood of the peak. The area of the neighborhood has to be small since it is the weak pumping. The neighborhood has to be around the peak with $x_1 \sim x_2$ since only around the peak the transmission coefficient is approximately one. As a result, we obtain immediately the pumped charge or pumped current of NS structure near the resonance is four times of that of corresponding normal structure. In the other extreme, for strong pumping, we take a large contour enclosing entire resonance line. Since Π^{NS} decreases much faster than Π^N away from the peak, it is understandable that the pumped charges (the integral of Π over the area enclosed by the contour) for both normal and NS structures are equal, which will be shown analytically below.

After the expansion in powers of δ in Eqs.(18) and (19) and keep the leading orders of δ, we have

$$F_3 = p^7[2+p(-1+z)](-1+z)^3(1+z)^4\frac{\delta^9}{64} \tag{24}$$

$$F_4 = 2(1-p)^2\delta^2 + [-\frac{1}{6}+\frac{2p}{3} + \frac{1}{2}p^2(-1+z^2)+\frac{1}{16}p^4(-1+z^2)^2]\delta^4 \tag{25}$$

So Eq.(3) becomes,

$$Q^{NS} = \frac{e}{\pi}\int_0^\infty pdp\int_{-1}^1 dz\frac{F_3}{F_4^3}\ \delta^2 \tag{26}$$

using the fact that $\lim_{\delta->0}\delta^5/(x^2+\delta^2)^3 = (3/8)\pi\delta(x)$, Eq.(26) becomes

$$Q^{NS} = 3\sqrt{2}e\int_{-1}^1 dz\frac{(1-z^2)^3(1+z)^2}{(1+6z^2+z^4)^{5/2}} = 2e \tag{27}$$

Hence the pumped charge for NS system is quantized at the same value of that of the normal structure.

Now we have a better physical picture for the transport properties of the NS structure. For the conductance or the I-V curve, we need S_{21} or $\mathcal{S}_{he}$. For normal structure, the current is given by $I^N = 2e/h\int dE[f(E-eV_1)-f(E-eV_2)]|S_{12}|^2$ and hence at resonance and at zero temperature $G^N = I^N/(V_1-V_2) = 2e^2/h$. For NS structure, we have[20,32] $I^{NS} = 2e/h\int dE(f(E-eV_1)-f(E+eV_1))|\mathcal{S}_{he}|^2$ and at resonance $G^{NS} = I^{NS}/V_1 = 4e^2/h$ which is the well known doubling of the conductance. For pumped charge or pumped current at resonance, however, it depends only on $\partial_{x_i}S_{11}$ or $\partial_{x_i}\mathcal{S}_{ee}$ ($E=0$ is assumed). Because of the constructive interference between direct reflection and multiple Andreev reflection in the *weak pumping regime*, the charge transfer increases by a factor of four when one of the lead becomes superconducting. In the strong pumping regime, however, the charge transfer is quantized at the value equal to that of normal structure, if the pumping contour is chosen such that the resonance line is enclosed. The physics behind this can be understood as follows. In the normal case, the contour enclosing the resonance line in the parameter space passes through the resonance line at two points $(x_1,x_2) = (0,-\delta)$ when the left contact is almost closed and $(-\delta,0)$ when the right contact is almost closed. When passing through those two points, the resonance level of the dot crosses the Fermi energy. At each crossing, the occupation of the level changes, and two electrons with opposite spin enter or exit the region between the barriers. Since one of the tunnel barriers has zero conductance at those points, it is clear that the electrons must have tunneled through the other contact upon entering or leaving the quantum dot. Hence, in the pumping cycle, electrons are shuttled pairwise through the dot. In the presence of superconducting lead, the resonance level (both the energy and the width) is exactly the same as that of normal case since the scattering matrix is given by $\mathcal{S}_{he} = i|S_{12}|^2/(1+|S_{22}|^2)$ when $E=0$. Therefore the same argument applies to the superconducting case and the quantization unit is $2e$. Note that our statement is only valid when the electron interaction is neglected. For the case of two normal-metal contacts, if interactions are included the quantization will remain, but now the quantum is only e: Only one electron at a time can enter the

region between the barriers; addition of a second electron is forbidden by Coulomb blockade. In the presence of superconducting lead, since the Andreev reflection requires two electrons with opposite spin in order to produce the supercurrent, it seems that the pumping is not allowed in the strong pumping regime due to Coulomb blockade. In this paper, we have also neglected the effect of temperature and the effect of inelastic scattering. As discussed in Ref. 17 the temperature will destroy the quantization of the pumped charge. When inelastic channel is present an additional physical mechanism for an incoherent pump effect will show up[33].

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FIG. 1. (a). The integrand Π of Eqs.(3) and (15) as a function of x_1 and x_2 for $\delta = -0.2$. For illustrating purpose, the origin of $\Pi^N(x_1, x_2)$ has been shifted by $(0.1, 0.1)$. (b). The cross-section of Π along the resonance line $x_1 + x_2 = -\delta$. Solid line: Π^{NS}; dotted line: Π^N. Inset: the cross-section of Π along the direction $x_1 - x_2 = c_0$ which is perpendicular to the resonance line. Left inset: $c_0 = 0.01$; right inset: $c_0 = -0.042$.

Fig1 a

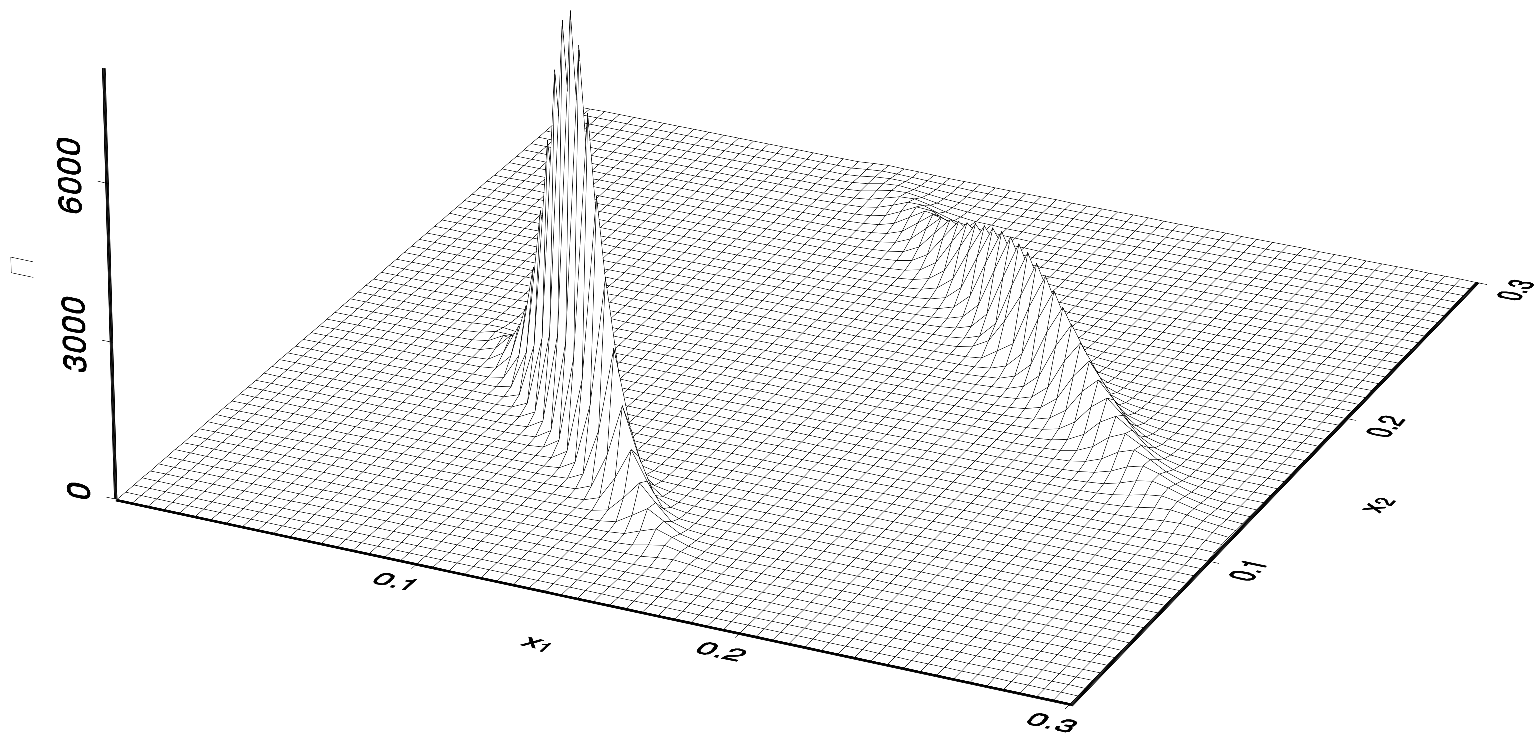
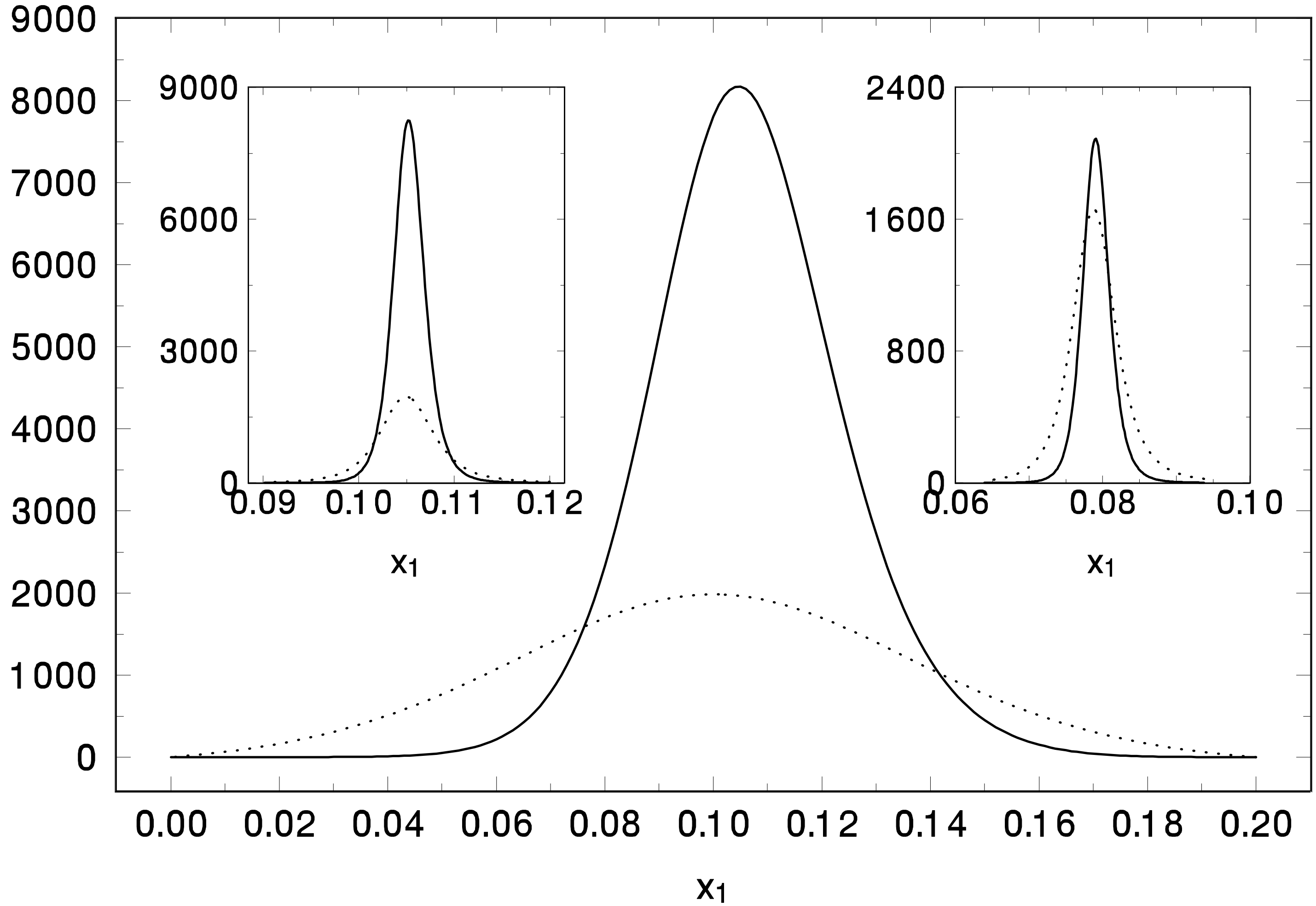


Fig1 b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